Proton elastic impact factors for two, three, and four gluons

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In this talk [1] we report on recent calculation of high energy baryon scattering amplitudes in QCD. Elastic baryon impact factors for two, three and four gluons are presented and their energy evolution is described that incorporates unitarity corrections. We find that the baryon couples directly to the BFKL Pomeron, to the BKP odderon and to a new state, a three-gluon BKP Pomeron. The new state may decay into four gluons with a new $3 \rightarrow 4$ transition vertex. This vertex defines the transition amplitude of the three-gluon BKP Pomeron state into two BFKL Pomerons.

1 Introduction

The complete picture of high energy scattering of hadrons in QCD has to incorporate effects of multiple scattering. In particular, when those effects are properly taken into account, they solve the problem of the rapid increase of Balitsky-Fadin-Kuraev-Lipatov (BFKL) [2] amplitudes with energy, that would eventually lead to a violation of the S-matrix unitarity. So far, the most successful realization of the unitarization of BFKL amplitudes was performed within the Balitsky-Kovchegov (BK) framework [3], for the scattering of deeply virtual photon, γ^*, on a large nucleus. This scheme relies on the dipole-like nature of the hard probe and on the large N_c limit. Unfortunately, the BK formalism is not sufficient to solve an important problem of the high energy baryon scattering. Baryons contain at least N_c constituents with a non-trivial color connection, and consequently the large N_c-limit for the baryon wave function is much more complex than it was for the γ^* (or a color dipole) projectile. This obstacle prohibited the direct solution of the baryon scattering problem within the s-channel approach [4].

Recently we attempted to analyze the baryon scattering amplitude within the t-channel framework [5], for $N_c = 3$. The formalism applied is derived in perturbative QCD and it resums to all orders the leading logarithmic contributions $(\alpha_s \ln(s))^n$ of the collision energy squared, s. It generalizes the Bartels-Kwieciński-Praszałowicz (BKP) evolution scheme [6, 7] valid for a fixed number of reggeized gluons in the t-channel, by inclusion of integral kernels that change the number of gluons in the t-channel [6]. In particular, in this approach the triple-Pomeron vertex was obtained [8] that was shown to match in the large N_c limit the vertex defining the BK equation. In this talk [1] results of the t-channel analysis of baryon scattering amplitudes are presented: the baryon impact factors and their small-x evolution are described.
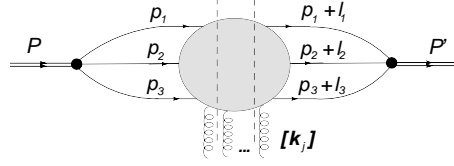
Figure 1: Baryon impact factor.

2 The impact factors

The baryon impact factors for n gluons are defined by multiple discontinuities of the elastic scattering amplitudes of baryons mediated by n-gluons, see Fig. 1. They are evaluated in the high energy limit, when only the leading power of the large light-cone momentum component is retained. The impact factors are given by the following formula,

$$B_{n;0}(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{l}_3) \;=\; I_{qq}^{(n)} \sum_{\text{diagrams}} F(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{l}_3)\; \mathcal{C}_n(\text{diagram}), \tag{1}$$

where the factor $I_{qq}^{(n)} \;=\; (-ig)^n$ represents the quark scattering amplitude (without the color factor). The overlap of the outgoing and incoming baryon wave functions is decomposed for each diagram into the color dependent factor $\mathcal{C}_n(\text{diagram})$, and the momentum dependent factor, $F(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{l}_3)$. Note, that the impact factor depends only on the total momentum transfers, $\boldsymbol{l}_1$, $\boldsymbol{l}_2$ and $\boldsymbol{l}_3$ to quark lines 1, 2, and 3 respectively. The color part of the baryon wave function is given by the fully antisymmetric tensor, $\epsilon^{\alpha\beta\gamma}$. Using the master formula (1) we performed the sum over all relevant diagrams and evaluated the baryon impact factors up to four external gluons, both in the C-even ($B_{n;0}$) and the C-odd ($\tilde{B}_{n;0}$) channel.

For two gluons, with transverse momenta $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, and color indices a_1 and a_2, only the Pomeron contributes. $B_{2;0}$ can be then represented as a sum of three pieces,

$$B_{2;0}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \delta^{\,a_1 a_2}\left[D_{2;0}^{\{1,2\}}(1,2)+D_{2;0}^{\{1,3\}}(1,2)+D_{2;0}^{\{2,3\}}(1,2)\right], \tag{2}$$

where we used a short-hand notation $D_{2;0}^{\{1,2\}}(1,2) \equiv D_{2;0}^{\{i,j\}}(\boldsymbol{k}_1,\boldsymbol{k}_2)$, $D_{2;0}^{\{1,2\}}(12,0) \equiv D_{2;0}^{\{i,j\}}(\boldsymbol{k}_1+\boldsymbol{k}_2,0)$, etc. In the components $D_{2;0}^{\{i,j\}}$ only quarks i and j scatter and the third quark is a spectator. All $D_{2;0}^{\{i,j\}}$ have the momentum structure of the color dipole impact factor, e.g.

$$D_{2;0}^{\{1,2\}}(1,2) \;\;=\;\; \frac{-g^2}{12}\left[F(12,0,0)+F(0,12,0)-F(1,2,0)-F(2,1,0)\right], \tag{3}$$

and similarly for $D_{2;0}^{\{1,3\}}$ and $D_{2;0}^{\{2,3\}}$.

For three gluons in the C-even channel, the impact factor can be decomposed into dipole-like components in an analogous way, $B_{3;0} = D_{3;0}^{\{1,2\}} + D_{3;0}^{\{1,3\}} + D_{3;0}^{\{2,3\}}$, and the dipole-like components have the reggeizing form known from the color dipole case,

$$D_{3;0}^{\{i,j\}}(1,2,3) \;=\; \frac{1}{2}\,g\,f^{a_1 a_2 a_3}\left[D_{2;0}^{\{i,j\}}(12,3)-D_{2;0}^{\{i,j\}}(13,2)+D_{2;0}^{\{i,j\}}(23,1)\right], \tag{4}$$

In the odderon channel, we see a distinct color-momentum structure,

$$\tilde{B}_{3;0}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \;=\; d^{\,a_1 a_2 a_3}\,E_{3;0}(1,2,3), \tag{5}$$

where for a symmetric function $F(1, 2, 3)$ and for $\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 = 0$, $E_{3;0}$ takes the form,

$$E_{3;0}(1,2,3) \;=\; \frac{ig^3}{4} \left[2F(1,2,3) - \sum_{j=1}^{3} F(j, -j, 0) + F(0,0,0) \right]. \tag{6}$$

For the case of four gluons ($C = +$) we see the further emergence of the gluon reggeization pattern for each of the dipole-like components, but in addition, a new structure, $Q_{4;0}$, appears:

$$B_{4;0} \;=\; D_{4;0}^{\{1,2\}} + D_{4;0}^{\{1,3\}} + D_{4;0}^{\{2,3\}} + Q_{4;0}, \tag{7}$$

where $D_{4;0}^{\{i,j\}}$ have the functional form known from the color dipole case, and

$$
\begin{aligned}
Q_{4;0} \;=\; & \frac{-ig}{2} \left[d^{\,a_1 a_2 b}\, d^{\,b a_3 a_4} - \frac{1}{3}\, \delta^{\,a_1 a_2}\, \delta^{\,a_3 a_4} \right] \left[\, E_{3;0}(12,3,4) + E_{3;0}(34,1,2) \,\right] \;+ \\
& \frac{-ig}{2} \left[d^{\,a_1 a_3 b}\, d^{\,b a_2 a_4} - \frac{1}{3}\, \delta^{\,a_1 a_3}\, \delta^{\,a_2 a_4} \right] \left[\, E_{3;0}(13,2,4) + E_{3;0}(24,1,3) \,\right] \;+ \\
& \frac{-ig}{2} \left[d^{\,a_1 a_4 b}\, d^{\,b a_2 a_3} - \frac{1}{3} \delta^{\,a_1 a_4}\, \delta^{\,a_2 a_3} \right] \left[\, E_{3;0}(14,2,3) + E_{3;0}(23,1,4) \,\right]. \tag{8}
\end{aligned}
$$

In the case of the odderon, for four gluons, one finds that the impact factor is fully exhausted by the reggeizing contribution, that is $\tilde{B}_{4;0}$ may be obtained from $\tilde{B}_{3;0}$ by all possible splittings of a single gluon into two elementary gluons with the color tensor f^{abc}, cf. Eq. (4).

3 The evolution

We demonstrated that the basic objects (modulo reggeization) defining the baryon impact factor are the dipole-like components, $D_{2;0}^{\{i,j\}}$, and the functions $E_{3;0}$ and $Q_{4;0}$. All these functions vanish if one of the gluon transverse momenta vanishes. They are also fully symmetric under permutations of the gluon momenta (Bose invariance). Thus, they are proper initial conditions for the BKP evolution: $D_{2;0}^{\{i,j\}}$ for the BFKL Pomeron, and $E_{3;0}$ for the BKP odderon spanned by three reggeized gluons. $Q_{4;0}$ may be interpreted as an initial condition for a C-even three-Reggeon state, where one of the reggeized gluons has the even signature.

In order to analyze the structure of unitarity corrections in the t-channel approach, one has to go beyond the Reggeon number conserving BKP equation, and include integral kernels describing splittings of $2 \to n$ Reggeons. Then, the small-x evolution of impact factors is given by a set of coupled integral equations with the initial conditions given by $B_{n;0}$ and $\tilde{B}_{n;0}$. We solved these integral equations for the baryon up to four gluons. In the odderon channel we found only the BKP evolution preserving the color-momentum structure of $E_{3;0}$. In the Pomeron channel the situation is more complex. The dipole-like pieces obey the BFKL evolution, preserving their color-momentum structure, but in addition a transition may occur to a four-Reggeon state, that may be projected on two BFKL Pomerons (Fig. 2). The amplitude of this transition is given by the $V_{2 \to 4}$ vertex (related to the triple-Pomeron vertex), well known from the analysis of γ^* scattering. In addition, we found the BKP evolution of the three-Reggeon state, Q_4. The state Q_4, however, may also decay into four
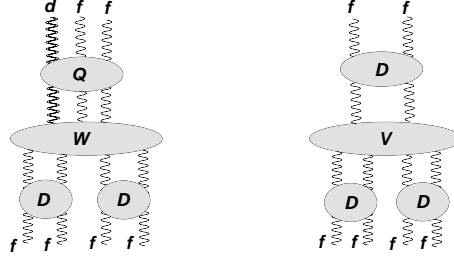
Figure 2: Transition vertices $W_{3\to4}$ (left) and $V_{2\to4}$ (right); reggeized gluons with the odd (f) and the even (d) signature are indicated.

Reggeons with the amplitude given by a new vertex, $W_{3\to4}$, that may be also be interpreted as a triple Pomeron vertex, but with the three-Reggeon BKP Pomeron, Q_4, that splits into two BFKL Pomerons, see Fig. 2. We point out that the possible direct two-Pomeron coupling to the baryon was not found. The lack of the direct two-Pomeron coupling, however, essentially relies on taking in account only the lowest Fock component of the baryon, and it holds only in the leading logarithmic $\ln(s)$ approximation.

4   Conclusions

We have analyzed the high energy scattering of a baryon projectile. The baryon, represented by three constituent quarks, was found to couple to the BFKL Pomeron, the BKP (three-Reggeon) odderon and a new state, a BKP Pomeron spanned on three Reggeons, out of which one has an even signature. The BFKL Pomeron may couple to one of dipole-like pieces of the baryon. Each dipole-like component of the baryon has the color-momentum structure of the genuine color dipole. The evolution of those states was analyzed up to four reggeized gluons in the t-channel. The dipole-like components were found to evolve in the same way as the color dipoles. Specifically, their evolution is driven by the BFKL equation, followed by a possible splitting of the BFKL Pomeron into four reggeized gluons (two Pomerons). The three-Reggeon Pomeron obeys the BKP equation and it may split into four reggeized gluons. This transition is driven by a new $3 \to 4$ reggeized gluon vertex.

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